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BOOK REVIEWS

Self Organised Criticality, Emergent Behaviour in Physical and Biological Systems. By H. J. Jensen Cambridge University Press, 1998. 153 pp. ISBN 0521 48371, £18.95.

I began to write this report on the day the economic dominoes cascaded. The Dow Jones tumbled over 500 points. The next day it had leapt almost 300 points in the other direction. Chance events? Or were the gods also signalling me that their messages were indeed written in the behaviour of sandpiles?

Theoretical scientists have a great yen for the supergeneral. The holy grail of physics is an all encompassing unified field theory which will explain the behaviours of all things big and small, from gravity to quantum effects. Several noted physicists claim they have already glimpsed the promised land and trumpet that the end of physics is nigh. Unfortunately they leave behind a trail of unexplained debris in addition to questions of a fundamental nature. We may indeed discover the elementary laws and particles of nature. But using these same simple building blocks to explain the variability of behaviours we observe in oceans and galaxies and zoos and the financial markets is quite another matter. The world is not one great bowl of porridge, dull and grey and uniform like crystals or simple gases, where everywhere is the same as everywhere else. It is rich in texture and we have to explain why. How does one account for planetary dynamics? How do we explain the sea spectrum, earthquakes of all sizes, the mind numbing complexity of landscapes and trees and species of plants and animals? How do we calculate the volume flux of water through a pipe under a large pressure head or the heat transported per acre per hour from ground to mountain top level at midday in the mid-latitudes? The challenge is to understand how systems of simple component parts behave as collective units when the parts are all coupled together into a strongly interacting ensemble and subjected to some overall external stress.

To fill this void, yet another supergeneral idea, called Self Organised Criticality, has been proposed.

The development of equilibrium statistical mechanics, associated with the names of Boltzmann and Gibbs, was one of the great successes of late nineteenth century physics. By calculating very simple functionals of the parameters involved in the rules of interaction, the collective behaviour of closed (no input, no output) systems of large numbers of interacting particles can be well understood. After perturbations, such systems relax to their respective thermodynamic (Bose–Einstein, Fermi–Dirac) equilibria and, in these states, the 'on the average' behaviour of 10^{23} interacting atoms can be calculated in terms of macroscopic quantities such as pressure and density and temperature. Except at very special values of various parameters at which the system undergoes a (second order) change of phase, local disturbances are felt only over certain characteristic distances called coherence lengths. Only at these special critical points do the coherence lengths diverge and correlations fall off algebraically rather than exponentially.

But many interesting systems are not closed and are not operating near some thermodynamic equilibrium. They are far from equilibrium, driven either directly or through some instability at one scale or place, and suffer losses at another. Often they exhibit no characteristic scale in space or time. Correlations decay algebraically rather than exponentially. Examples of such systems are widespread: the Gutenberg–Richter law of earthquakes whereby, over many decades, the frequency of occurrence is inversely proportional to size; Raup's empirical law on the frequency of extinction rates; Zipf's law on the sizes of cities or the frequency of the words used in literature. All of these belong to that large set of systems which exhibit the ubiquitous $1/f$ noise.

It was an attempt to come to grips with the phenomenon of $1/f$ noise that led Bak, Tang & Wiesenfeld (BTW) in 1987 (*Phys. Rev. Lett.* vol. 59, pp. 381–384) to propose the idea that slowly driven systems far from equilibrium have a tendency to wind themselves up and, without direct external tuning, self organise themselves into states which resemble the critical states at phase transitions of equilibrium systems. Precariously perched at these precipices in one of many marginally stable states, they are extraordinarily vulnerable. Very small local causes can trigger spasms of events of all durations and sizes. It is as if the whole world were teetering on the edge of some giant orgasm where the slightest stimulus could cause the release of an enormous build-up of tension with uncontrollable avalanches of giggling and laughter, sneezing fits or whatever. You can all let your imaginations run completely free at this point!

BTW built models to simulate and test this picture. They began with a two-dimensional array of coupled pendula, each connected to four neighbours by torsion springs so that via the twist in the connecting coils, the rotation of one pendulum would cause rotation in those at neighbouring sites. At first if you gave a particular pendulum a good swing so as to loop-the-loop, its influence on quiescent neighbours would be dispersed because of sharing, so that, while they would swing a lot themselves, they would not have enough energy to follow the original site in going over the top. But as the system is wound up, as more and more sites are given loop-the-loop perturbations, the probability that one pendulum going over the top will cause successive neighbours to do the same becomes higher. BTW counted the number of loop-the-loop events which followed each additional loop-the-loop perturbation at a single randomly chosen site. The probability density $P(s)$ exhibited power law behaviour.

A subsequent model, which has since become the granddaddy of paradigms for such systems, investigated the avalanches that occur in connection with building sandpiles. The sandpile automaton is simple to explain. Begin with a, say two-dimensional, lattice of sites and at given intervals add a grain of sand at a randomly chosen site. Look at the configuration after each addition. If the site to which the addition was made has become supercritical, say it has now four grains, then distribute one grain to each of its edge sharing neighbours. For a site at an edge border of the lattice, one grain of sand is forever lost and at a corner two are lost. Now, when you begin the process, it is not likely that a supercritical site will have a neighbour which will also become supercritical but one can see that as the process of grain addition continues and as the system gets wound up, this probability increases. If a neighbour should be supercritical, then continue to redistribute and so on. The process of redistribution is called a toppling and a sequence of topplings is called an avalanche. One continues to add grains and allow the relaxation to occur until one reaches a statistically steady state after which the histogram of the frequency of events (number of topplings caused by a single addition) becomes stationary.

The power law response of the sandpile automaton led BTW to propose that the behaviour they observed was universal and had manifestation in earthquakes, landscapes and biological extinctions. Slowly driven ensembles of strongly interacting systems with simple local laws asymptote (self organise) without outside tuning to an extremely vulnerable critical state where minor and local disturbances can give rise to avalanches of intermittent events of all sizes and durations with no dominant scales. They called this behaviour Self Organised Criticality (SOC).

The notion proved to be intriguing and had immediate appeal, gaining the attention of many colleagues. In the past ten years, around 2000 papers have been published on the topic in a wide variety of journals. Again, as in high energy physics, the mad rush probably reflects our innate attraction for the seductive siren of an inspirational theory of everything. Nevertheless, the idea has generated much interest and in the minds of some has reached that stage of maturity where its dissemination deserves the book format.

There are two such efforts to date, each of which has just appeared. Per Bak himself has written a very readable and enjoyable account ambitiously (outrageously?) titled *How Nature Works* (Oxford University Press, 1997). It is the kind of book designed to capture the scientifically minded air traveller who has already been through *A Brief History of Time*, and several of the series on the Science of Consciousness. It is homespun in its language and extravagant in its claims that Self Organised Criticality is a unique and new scientific insight that explains all manifestations of intermittent behaviour. Such accounts are often enjoyable precisely because they are bold, stimulating and thought provoking. At the same time, the discerning reader has learned to treat their conclusions with a sensible degree of caution.

The second book, written by Henrik Jeldtoft Jensen of Imperial College is published by – who else – Cambridge University Press and has more modest and academic goals. Its preface states that it is intended for everyone interested in one of the most exciting and ambitious current developments in the field of physics and complex systems. It suggests that a little bit of mathematics might be handy now and then. Its foreword claims that it would be an ideal textbook for graduate students.

The introduction sets the stage and, for the most part, does it well. The author begins by raising the question as to whether strongly interacting systems of many components driven by external sources can develop any kind of universal behaviour. He introduces the BTW hypothesis that many such systems self organise to a set of marginal states from which precarious perches small stimuli can lead to large and unpredictable responses characterised by power law statistics and an absence of specific scales. He reviews equilibrium statistical mechanics and the rules of that game and reminds us of the special behaviour of such systems near certain critical values of the parameters. He then purrs the seductive message that self organisation plus criticality equals complexity. However, he is careful to stress that to date there is no accepted definition of SOC nor any clear delineation of the circumstances under which one can expect SOC type behaviour to occur. The introductory chapter is followed by a short and useful discussion on distributions, correlations and their power spectra. He points to the fact that an $f^{-\beta}$ power spectrum is equivalent to $\tau^{-1+\beta}$ temporal correlations and therefore to ultra-slow decay as β approaches one, and to the connection of the SOC behaviour with the random superposition of independent Poisson processes.

The meat of the book is contained in Chapters 3–5. Chapter 3 reviews a cross-section of systems which exhibit or are candidates for SOC behaviour. He discusses sandpiles (where inertia effects dominate for the most part and lead to periodic oscillations between some average relaxed slope and maximum ‘critical’ slope), rice-piles (long grains with lots of intergrain friction which do exhibit SOC behaviour), the flux of magnetic vortex lines across the walls of a superconducting tube and droplet formation, both of which would appear to exhibit SOC behaviour. He then discusses briefly the evidence for, and the controversies surrounding, the inclusion of earthquakes and species evolution as SOC events.

Chapter 4 deals with computer models. It introduces several automata, one- and d-dimensional sandpiles, discusses the notion of the set of marginally stable configurations, the automata for non-conservative earthquake models with both nearest neighbour and random neighbour (which destroys spatial correlations and is more accessible to mean field theories) interactions. He also comments on the role of local dissipation, whereby there is a net loss associated with each toppling event, and demonstrates the presence of a phase transition from SOC to non-SOC behaviour as the local dissipation is increased. He connects the results of the OFC earthquake model (Olami, Feder & Christensen) to earlier models for stick-slip processes introduced in the sixties by Burridge and Knopoff. In addition, he includes discussions of lattice gas models, forest fire models, interface growths and evolution models. This chapter gives a very readable account of the issues and is supported by appendices giving explicit codes.

Chapter 5 is an attempt to supply the real beef. Having whetted our appetites, the author attempts to construct a theoretical foundation for systems which exhibit SOC behaviour. Three of them, the mean field approach, a Langevin description and the renormalisation group approach which successively coarsegrains the automaton, are fairly standard for most modern physicists. For each the explanation is adequate but an instructor would need to fill in a lot of material in order to make the ideas accessible to graduate students. There are several annoying but not serious typos. q_c should be replaced by $1/q_c$ in a number of places on page 84. I also fail to understand the pedagogic value or importance of 5.2.3, which includes spatial effects without a clear purpose. Exercises without answers, or at least hints, are anathema to me. This is a serious problem with the book as a teaching tool. While giving lots of ancillary information in appendices, the author often fails to include a more careful treatment of some important calculations.

The new material of Chapter 5, or at least the material which is likely to be new to many in the profession, is the introduction of the exactly solvable models of the Abelian Sandpile developed by Deepak Dhar and several colleagues. This is new and important stuff. Exactly solvable models are valuable. The solutions of the Ising model and N-vertex models of equilibrium statistical mechanics, associated with the names of Onsager, Baxter & Sato, Miwa & Jimbo, led to a great deal of new understanding about near critical behaviour. Likewise the exactly solvable soliton equation greatly enhanced our knowledge of infinite-dimensional Hamiltonian systems.

The Abelian Sandpile is defined by the specification of a ‘toppling’ square $N \times N$ matrix Δ, N the total number of sites, which contains all the information as to what are the critical heights on each site j and the redistribution rules when a given height has reached critical. Abelian connotes the independence of the dynamics of the order the rules are applied. The toppling information at site j does not depend on other sites. Jensen explains the new ideas very well: the set-up, the rules, the notion of the recurrent set R in the statistically steady state, the augmented set A of configurations reachable from R by the addition of grains, the notion of configuration equivalence and the use of this in calculating the size of R. I would have liked more of this. Concrete examples (say with $N = 4$ with Δ given by the discrete Laplacian) would have been welcome as would some discussion of the toppling invariants. As a supercritical configuration relaxes through other configurations to its final state in R, certain linear functionals of the site heights are conserved (modulo unity). These can be used to characterise R. Also it would have been good to see how one calculates the power law exponents for the probability distribution of avalanche sizes if that is possible.

The final chapter addresses the questions which will have probably vexed the reader the whole way through. The author has told us about the notion of SOC, characteristic behaviours associated with the avalanche of all sizes phenomenon, produced a bunch of 'looks like' evidence, and given us lots of computer games to play with. But he has not told us what SOC actually is. The reason for this omission is of course that there is no accepted definition of SOC. Nor is there a delineation of the properties with which a system must be endowed to exhibit the type of behaviour that falls under the SOC umbrella. Admittedly, precise statements are difficult but I would have been partly mollified with the kinds of explanations soliton theorists give for the ubiquitous appearance of integrable models such as the Korteweg–de Vries or nonlinear Schrödinger equations as asymptotic descriptions for a wide variety of physical phenomena characterised by certain, *a priori* identifiable, properties. I find the author's attempts to do this to be rather lame. He tells us that systems have to be strongly interacting but when one uses words such as strong or weak one should also say with respect to what. Presumably one might find a measure of the ratio of interparticle coupling strength to the strength of the particle's individual dynamics. Elongated rice grains make better SOC sandpiles because their intergrain friction overcomes their inertia. We are told threshold is important but thresholds are important in lots of contexts. We are told about tuning and the importance of tuning without direct outside influence. While I did not expect a global stability statement on the self organised critical state, a little more discussion of local stability might have been helpful.

I missed also discussion of connections with the appearance of self organisation and power law behaviour in other systems such as pattern formation and turbulence where there are more concrete theories to explain these behaviours. Also the book makes no mention of how the complex behaviour expected of SOC systems makes contact with the unpredictable behaviour of the so-called chaotic dynamical systems.

Nevertheless, I must congratulate the author on taking up the challenge of trying to bring some exciting, novel, albeit controversial ideas, to a more general audience. All in all, I enjoyed the book and found the effort of reading it stimulating and worthwhile. But we should remain sanguine about the subject itself. No more than catastrophe theory has explained all sudden transitions, than chaos theory has explained fully developed turbulence, will the notion of self organised criticality explain all complexity.

A. C. Newell

Particle Image Velocimetry. A Practical Guide. By M. Raffel, C. Willert & J. Kompenhans. Springer, 1998. 253 pp. ISBN 3540 63683 8. £49.00.

Particle Image Velocimetry. A Practical Guide is an attempt at compiling, sometimes in a rather brief form, most aspects of the experimental fluid velocity measurement technique which has become known as Particle Image Velocimetry (PIV). In the authors' own words "The intention of this book is to present in a more general context mainly those aspects of the PIV technique relevant to the applications". In most aspects of PIV the authors have achieved this stated aim and the result is a useful reference for the experienced user and a good introduction for the novice. The relevant aspects of PIV are explained in this book in a succinct form and much of the information provided will assist novice users to avoid beginners' errors. This book also helps to de-mystify a number of the misconceptions regarding the complexity of setting up one's own PIV system which are often promulgated by suppliers of PIV "black-box" hardware and software. The basic principles are conceptually simple, but their successful application requires rigour in both the experimental phase and the PIV processing. This practical guide provides sufficient detailed information so that even novice users can set up a complete PIV measurement system without resource to "specialist PIV equipment suppliers".

This book has eight chapters of which the last is entirely devoted to applications of PIV with full experimental and PIV processing details provided. Although all the seminal papers are referenced in the bibliography, there is a heavy emphasis on the work published by the authors and their colleagues in associated laboratories. However, the authors are honest about their bias and refer the reader to some recent reviews which cover a much more extensive literature.

The first chapter provides a brief historical background to PIV. Chapter 2 introduces the different aspects of the experimental components. The chapter starts with the tracer particles, their mechanical and optical properties, and gives examples of how to generate appropriate tracer particles. This is followed by a discussion of the techniques for illumination, including continuous and pulsed lasers and white light sources. The optics to form light sheets are introduced as well as photographic and digital recording. There is an emphasis on the latter in this book. Chapter 3 introduces the mathematical background of the image processing component of PIV. Chapter 4 is devoted to recording techniques, and the operation and typical characteristics of all types of CCD sensors currently on the market are presented and their relative advantages and disadvantages are discussed. Single-frame and multi-exposure recording, image-shifting and its implementation are also considered.

Chapter 5 presents the practical implementation of the image processing methods discussed in chapter 3. Optical evaluation as it was originally used for multi-exposure single recordings is discussed with its more modern digital auto-correlation counterpart, the latter acting as an introduction to the fully digital cross-correlation analysis of two single-exposure recordings and multi-exposure with image shifting. Various improvements such as sub-pixel peak detection are discussed and there is a guide to the selection of optimal experimental and image analysis parameters to obtain accurate PIV measurements. The effects of background noise, velocity gradients and out-of-plane motion on the accuracy of PIV are also discussed in the chapter. Chapter 6 is entirely devoted to the post-processing of the velocity measurements, beginning with data validation, followed by data replacement and interpolation. The estimation of differential and integral quantities from the measured velocity field and the uncertainties associated with determining these quantities are also covered. Although this chapter is a good introduction to the post-processing and uncertainty analysis of measured velocity fields, the serious practitioner should also refer to recent published work where more up-to-date results are presented.

Three-component PIV measurement in planar domains is the topic of chapter 7. Half this chapter is devoted to stereo imaging PIV in which reconstruction geometry, image dewarping, calibration procedures and error estimation are presented and discussed. The second part deals with dual-plane PIV to determine the three velocity components in a plane. The last chapter of this book introduces a number of examples of application of PIV in gas and liquid flows. These examples are generally well documented with all pertinent experimental and image processing parameters specified. Applications with both photographic and digital image recording are presented.

This book is an important contribution to the literature of fluid velocity measurement techniques, but there are a number of criticisms that I have to make. The most serious one is editorial in nature: many of the references in the bibliography and their referencing numbers in the text are incorrect. In some instances there is not even a reference number, but only a [?]. These errors can be very disconcerting and could lead readers to the wrong reference. In my opinion, these bibliography errors reduce the value of this book as an authoritative reference on the PIV technique.

The other criticism is technical in nature and relates to a number of the documented experiments in chapter 8, specifically to those that use oil particles with a diameter of approximately 1 μm to seed the air flow. Even taking diffraction-limited imaging into account the particle images are always less than the equivalent of one pixel in the image plane. In other words there is not sufficient spatial resolution in the image plane, even for photographic recording, to resolve these particle images. As the optimum particle diameter is given in section 5.5.2 of this book as between 2 and 3 pixels, there is an inconsistency, but the authors make no comment on this.

In summary, these two criticisms aside, the book is a good reference on the PIV technique for the experienced practitioner, as well as the novice user. The technique is complementary to hot-wire anemometry and laser Doppler anemometry among the available fluid velocity measurement techniques and, as such, this book should find its place along with complementary books on those techniques on the bookshelf of the experimental fluid dynamicist.

J. Soria

Explosive Instabilities in Mechanics. By B. Straughan. Springer, 1998. 196 pp. ISBN 3-540-63589-0, $59.95.

The question whether solutions exist globally in time or develop singularities in finite time has always been a major focus in the study of partial differential equations. This book is concerned with examples where finite-time blow-up, or at least very rapid growth, occurs in mechanical systems, and primarily those of fluid mechanics.

The book begins with a discussion of basic techniques to show finite-time blow-up in nonlinear parabolic differential equations and systems and then proceeds to discussing a variety of examples. The remaining two thirds of the book are concerned with equations motivated by mechanics. The problems surveyed include the open question of finite-time blow-up for the Euler and Navier–Stokes equations, the development of infinite temperature gradients in Bénard–Maragoni convection (in the limit of infinite Prandtl number), blow-up backward in time for nonlinear viscous fluids, flows of second-order fluids, generalized KdV equations, ferrohydrodynamics, temperature blow-up in ice sheets, blow-up in Volterra equations resulting from combustion problems, chemotaxis, Hadamard instabilities in granular flows, sea ice dynamics and flows with pressure-dependent viscosity, and the rapid growth of instabilities in parallel shear flows at high Reynolds number.

This is a large number of topics to be discussed in a book of this length, and the exposition of each topic is necessarily brief. The style of the book is essentially similar to a review article, summarizing work that has appeared in the literature. I believe the book will be valuable as a survey and a reference. The list of topics is by no means exhaustive for blow-up in mechanics (as the author himself points out in the introduction, the whole field of development of shock waves has been omitted, and there are other examples which could be cited). Nevertheless, the selection of topics which have been included is quite broad, and the book should be of interest to readers from many different subdisciplines of fluid mechanics. The emphasis throughout is on mathematical results; little effort is made to assess the physical consequences of the results or the relevance of the models.

The book would have benefited from some more exposition and systematic organization. For instance, in Section 3.2, the issue of blow-up for the Euler equations is first introduced as an open problem, and then examples of blow-up are given, and it is never pointed out what the difference is! (In the examples, the initial condition is unbounded at infinity, which is not allowed in the 'classical' version of the problem.) The mathematical introduction is focused on nonlinear parabolic equations with polynomial nonlinearities, and not all the problems discussed in the rest of the book are of this type. In some instances, I find it hard to see what, if anything, problems grouped in the same chapter have in common.

References to the literature are extensive and, indeed, are a crucial component of the value of this book as a resource. Nevertheless, there are some omissions, for instance in the final section where the book gives the appearance that the study of eigenvalue spectra for two-layer viscous shearing flows is just beginning, when in fact numerical calculations using the Chebyshev-tau method have been in widespread use for the past fifteen years, and asymptotic studies of limiting cases go back as far as 1967.

M. Renardy